ON THE ANGLE OF INCLINATION OF THE EQUIVALENT LIGHTNING CHANNEL

H. BHATTACHARYA and MANORANJAN RAO

Department of Pure Physics, Calcutta University

(*Received May* 2, 1964)

ABSTRACT. It is shown that in the case of waveforms of reflected type of comparatively near origin, an estimate of the angle of inclination of the equivalent lightning channel can be made by measuring the relative amplitudes of the successive orders of reflection. The calculations have been carried out for four typical reflected types of the waveforms. Some of the characteristics of these waveforms like the phase-reversal, have been accounted for.

INTRODUCTION

The relative amplitudes of the sky pulses successively reflected from the ionosphere have been shown to give an estimate of the ionospheric reflection coefficient (Schonland, *et al.*, 1940). Since the radiation from the return-stroke channel can be regarded as emitted from a series of linear channel elements, one above the other, each inclined to the horizontal at a different angle, the obseved pulse will be the resultant of the pulses from these elements. The contribution of each of the pulses will depend upon its angle of emission. Thus, in principle, it should be possible to calculate the angle of inclination of the equivalent lightning channel from an analysis of the observed waveforms.

THEORETICAL CONSIDERATIONS

If θ is the angle made by the channel element with the horizontal (Fig. 1A), ϕ the angle with the horizontal at which the radiation is emitted and γ be the amplitude of the elementary ground-pulse emitted by it at an angle $\phi = 0$ then the corresponding amplitude σ_n, of its contribution to the n-th sky-pulse would be given by

$$\sigma_n = \frac{\sin \overline{\theta - \phi_n}}{\sin \theta} \cdot \gamma \tag{1}$$

where ϕ_n is the value of ϕ for the n-th sky-pulse. Equation (1) may pass from positive to negative as ϕ_n passes through the angle θ, at which poins $\sigma_n = 0$. If $\theta > \pi/2$ the ratio $\frac{\sin \overline{\theta - \phi_n}}{\sin \theta}$ is always positive. It can be seen from equation (1) that when a sum of all such elementary pulses is made we may expect to find considerable variation in the resultant form of the pulse with the order of reflection n. However, such variations will not be prominent in the low-order reflected pulses from distant sources since the lightning channel is usually vertical and since ϕ_n is such cases is small. Figure 2 shows that the low-order pulses are very similar

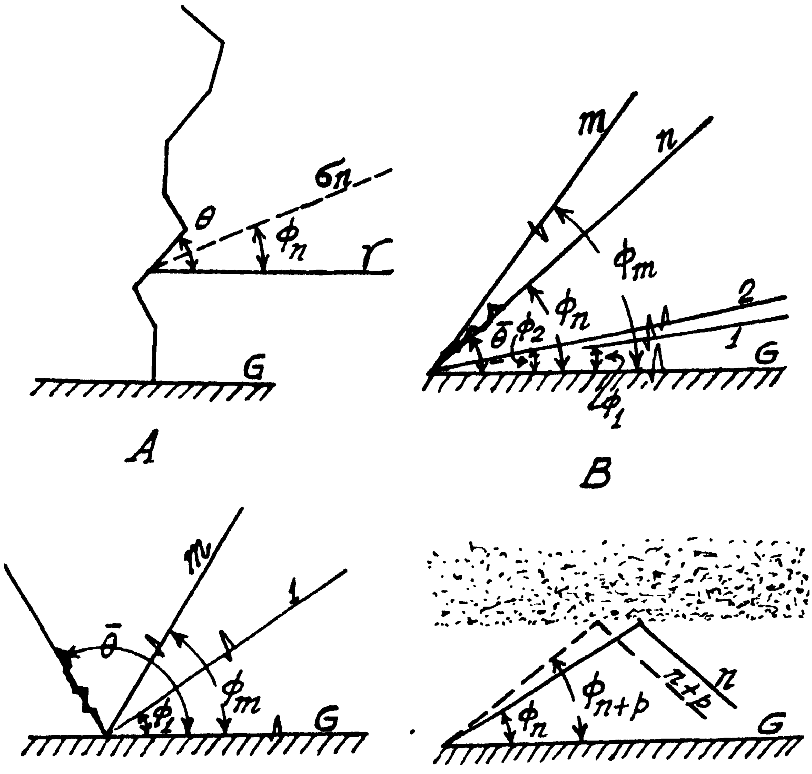

Fig. 1. (Taken from Schonland *et al*, 1940).

Fig. 2. May 27, 1963. 1800 IST.

Fig. 3. June 6, 1963. 1620 IST.

in form to the ground pulse. With nearer sources or with pulses of higher order at any distance, there may be a considerable change in the final form of the pulse. A typical oscillogram showing the variation of pulse form with the order of reflection is shown in Fig. 3.

If now the actual channel, made up of a series of elementary channels, one above the other, and making different angles with the horizontal, be replaced by an equivalent channel making an angle $\bar{\theta}$ with the horizontal, then, for $\bar{\theta} < \pi/2$, a reversal in phase will occur for a value of $\phi_n \geqslant \bar{\theta}$ and the amplitude of the sky pulse will pass through a minimum at $\phi_n = \bar{\theta}$ (Fig. 1B). Figure 4 is a typical example of such type. For $\bar{\theta} > \pi/2$ the radiation of the channel, as a whole, may be greater in the direction of the sky pulses than along the ground (Fig. 1C). A typical illustration of such type is Fig. 5.

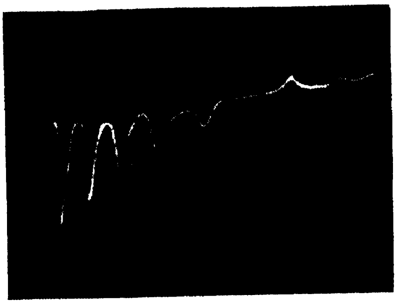

Fig. 4. June 6, 1963. 1620 IST.

Fig. 5. June 26, 1963. 1650 IST.

In what follows is given a possible method of estimation of $\bar{\theta}$ depending upon the relative amplitudes of the sky pulses and also the distance of the source and ionospheric reflection height.

Following Schonland *et al.* (1940) if s_{n+p} and s_n are the amplitudes of the $(n+p)$th and n-th sky pulses, produced by the summation of σ_{n+p} and σ_n, the ratio of the recorded amplitudes S_{n+p} and S_n will be given by

$$\frac{S_{n+p}}{S_n} = \frac{(s_{n+p}/D_{n+p})\cdot(r_i)^{n+p}\cdot(r_g)^{n+p-1}\cdot\cos\phi_{n+p}}{(s_n/D_n)\cdot(r_i)^n\cdot(r_g)^{n-1}\cdot\cos\phi_n}$$

or

$$\frac{S_{n+p}}{S_n} = \frac{s_{n+p}}{s_n}\cdot\frac{D_n}{D_{n+p}}\cdot\frac{\cos\phi_{n+p}}{\cos\phi_n}\cdot(r_i\cdot r_g)^p \quad \ldots \quad (2)$$

where D_{n+p} and D_n are the group paths for the $(n+p)$th and n-th resultant sky pulses and r_i and r_g are respectively the reflection coefficients of the ionosphere and the earth. If ϕ_{n+p} and ϕ_n be the angle made by $(n+p)$th and n-th pulses with the horizontal, then it can be shown (Fig. 1D) that

$$\frac{D_n}{D_{n+p}} = \frac{\cos\phi_{n+p}}{\cos\phi_n}$$

Therefore,

$$\frac{S_{n+p}}{S_n} = \frac{s_{n+p}}{s_n}\cdot\frac{\cos^2\phi_{n+p}}{\cos^2\phi_n}\cdot(r_i\cdot r_g)^p \quad \ldots \quad (3)$$

Following equation (1) we have for the equivalent channel inclined to the horizontal at an angle $\bar{\theta}$:

$$\frac{s_{n+p}}{s_n} = \frac{\sin \overline{\bar{\theta}-\phi_{n+p}}}{\sin \overline{\bar{\theta}-\phi_n}} \qquad \dots (4)$$

From equations (3) and (4) we have

$$\frac{S_{n+p}}{S_n} = \frac{\sin(\bar{\theta}-\phi_{n+p})}{\sin(\bar{\theta}-\phi_n)} \cdot \frac{\cos^2\phi_{n+p}}{\cos^2\phi_n} \cdot (r_i \cdot r_g)^p \qquad \dots (5)$$

or

$$\frac{S_{n+p}}{S_n} = \frac{\tan\bar{\theta}-\tan\phi_{n+p}}{\tan\bar{\theta}-\mathrm{ant}\,\phi_n} \cdot \frac{\cos^3\phi_{n+p}}{\cos^3\phi_n} \cdot (r_i r_g)^p \qquad \dots (6)$$

However, if the channel is nearly vertical ($\bar{\theta} \simeq \pi/2$) and both the order of reflection n and the distance of the source D are large, then physically speaking, S_{n+p}/S_n will not be far from unity. Further, under such circumstances $\cos\phi_{n+p}/\cos\phi_n$ will approximate to unity. If then we take $r_g = 1$, as is the case for long radio waves, equation (5) reduces to

$$r_i \geqslant \sqrt[p]{S_{n+p}/S_n} \qquad \dots (7)$$

This equation should give a lower limit to r_i which is not very far from its true value. Evidently $\bar{\theta}$ can be calculated from equation (6) taking $r_g = 1$.

CALCULATIONS

For the evaluation of the angles ϕ_{n+p} and ϕ_n, the height h at which the atmospheric pulses are reflected from the ionosphere and the distance D of the lightning discharge are approximately calculated graphically, following Caton and Pierce (1952), from the equation given by :

$$ct_n = \sqrt{D^2 + 4n^2h^2} \qquad \dots (8)$$

where t_n = time interval between the emission of the primary pulse and the arrival at the receiver of the pulse that has undergone n reflections at the ionosphere,

$D \approx$ distance between the source and the receiver,

n = order of reflection,

h = ionospheric reflection height,

c = velocity of light.

Two sets of graphs, based on expression (8) have been drawn : (1) showing time-intervals between the first sky pulse and successive sky pulses, against D for a constant value of h and (2) showing similar time intervals against h for a constant value of D(see Figs. 6 and 7). A reference pulse is selected on the waveform and the time-interval between this and the succeeding pulses of the same sign are measured. Trial values are then adopted for h and the reference pulse order, these being varied until a constant distance corresponding to all the measured

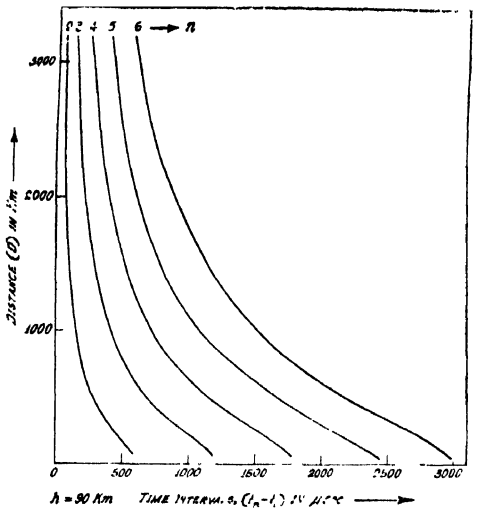

Fig. 6

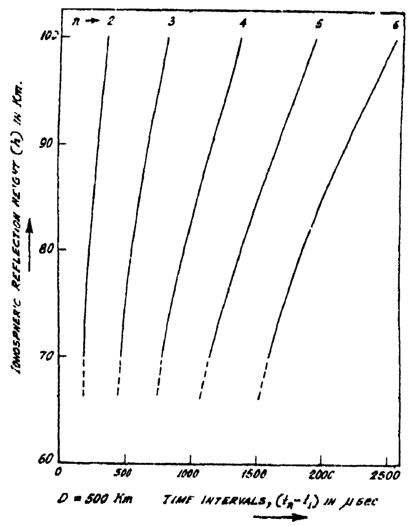

Fig 7

time-intervals is derived from set (1). Then a check is made on the obtained values of h and D by means of the set of graphs (2) by following a similar procedure. This then represents a mean determination over the waveform of the distance D and ionospheric reflection height h.

The values of D and h for the four oscillograms shown in figures 2 to 5 are as follows.

TABLE

Fig. No.	Distance D in km	Height h in km	$\bar{\theta}$
2	300	88	112°
3	180	93	109°
4	480	80	59°36′
5	375	92	148°

The values of r_i have been calculated taking $p = 2$ from equation (7) for each of the oscillograms. Taking r_g as unity and r_i as 0.8, $\bar{\theta}$ is calculated from the equation (6). The values of $\phi_{n+p}(\phi_{n+2}$ in the present case) and ϕ_n have been obtained from the computed values of D and h for each of the waveforms.

The calculated values of the inclination θ of the equivalent lightning channel are shown below the respe tive oscillograms and are als listed in last column of Table I.

DISCUSSION OF THE RESULTS

Referring to Figs. 2 and 3 it can be readily seen that though, both of the waveforms give nearly same values for $\bar{\theta}$ and have the same order of reflection, the change in shape of the latter is more pronounced, as it originated in a nearer source. This, thus, is in accordance with what has already been discussed earlier. The value of $\bar{\theta}$ in the case of Fig. 4 is 59°36′ which by being less than 90°, causes a reversal in the phase of the pulse after certain orders of reflection. Thus the amplitude of the pulse undergoes a minimum at $n = 5$ in the waveform of the Fig. 4 and a reversal of phase is obvious at $n = 6$. Finally, the waveform in Fig. 5 shows a ground pulse height smaller than the first order of reflection. The calculated value of $\bar{\theta}$, in this case, is 148°. Considering the group paths for the ground pulse and the first order of reflection and also the value of ϕ_1, calculated from D and h, it can be shown that the ratio of the amplitudes of the first order pulse to the ground pulse is greater than unity, the value of r_t being taken as 0.8. Thus the observed nature of the waveform can be accounted for.

An additional check can be made on the calculations in the case of the waveform given in Fig. 4. As already explained above, the minimum of the amplitude occurs when $\phi_n = \theta$. Here the minimum occurs when $n = 5$ and if ϕ_5 be evaluated, it should be of the same order as θ. In fact, taking the distance of the lightning source as 480 km and the ionospheric reflection height as 80 km, ϕ_5 comes out to be between 59° and 59°6′ which agrees fairly well with the calculated value of $\bar{\theta} = 59°36'$.

CONCLUSION

The angle of inclination of the equivalent lightning channel can be obtained from the measurement of successive amplitudes of the reflected-type of waveform except in the case of the smooth sinusoidal type (Budden, 1951, 1952) which can explained on the basis of the mode theory of propagation.

ACKNOWLEDGMENT

The authors wish to express their grateful thanks to Prof. S. R. Khastgir, D.Sc., F.N.I., for his valuable guidance throughout the progress of the work. They also thank the C.S.I.R. for sponsoring the research scheme on Atmospherics.

REFERENCES

Budden, K. G. (1951), *Phil. Mag.*, **42**, 833.

" " (1952), *Phil. Mag.*, **43**, 1179.

Caton, P. F. and Pierce, E. T. (1952), *Phil. Mag.*, **43**, 393.

Schonland, B. F. J., Elder, J. S., Hodges, D. B., Phillips, W. E. and van Wyk, J. W. (1940), *Proc. Roy. Soc.* ***A*176**, 180.